Pre-Health Post-Baccalaureate Program
PHY2053 Study Guide & Practice Problems

Topics Covered:

Position, Velocity, and Acceleration
1-Demensional Motion
Ramps
Relative Velocity
Projectiles

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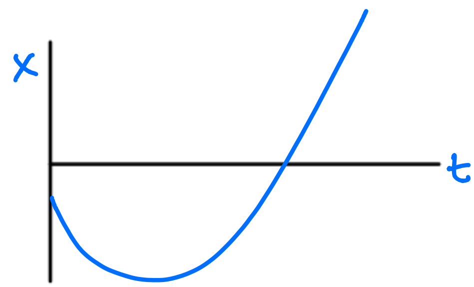
Position, Velocity, and Acceleration

1. Position (m)
    a. Position vs. time graph examines the distance than an object has moved or travelled (m or km) during the duration of the trip (s, min, or hr).
    b. Position is denoted by *x* and is graphed on the y-axis.
    c. Time is denoted by *t* and is graphed on the x-axis.
    d. The graph is not a picture of the motion! It instead models distance away from the starting point (origin) at various points in time.
    e. Slope of the curve is the velocity, *v*.



2. Velocity (m/s)
    a. Velocity can be defined as the distance travelled over a set amount of time, which is modeled by the equation:
    $$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$
    b. "Instantaneous velocity" refers to an object's speed and direction at a specific point in time. When we use the term "velocity," this is typically what we mean.
    c. "Average velocity" refers to an object's average speed and direction over the entire trip.
    d. Velocity is a vector quantity, as it has both magnitude and direction.
    e. Speed is a scalar quantity (and equal to the magnitude of the vector quantity), as it only has a magnitude but no direction.
    f. Because $t_f > t_i$, t will always be a positive value. The direction of velocity, then, is determined by the sign of $x_f - x_i$. If the value of $(x_f - x_i) > 0$, then the velocity is positive. If $(x_f - x_i) < 0$, however, then the velocity is negative.
    g. Recall that the slope of a curve can be written as:

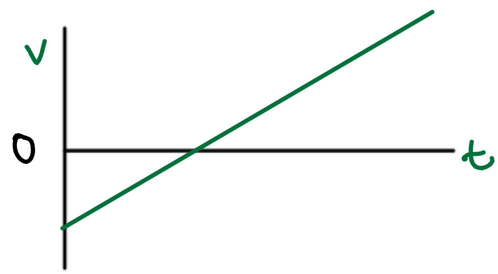
$$V = \frac{dx}{dt}$$

$$slope = m = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

On the position vs. time graph, the "rise" (or y-axis) is *x*, and the "run" (or x-axis) is *t*. The slope of the curve, then, is the velocity of the object at that point in time.

h. Using the position vs. time graph to determine the velocity of the object at any given point in time, we can now create a velocity vs. time graph.

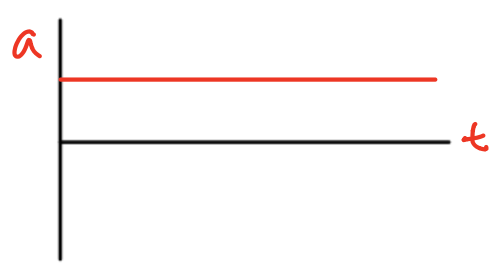
i. We can also go from a velocity vs. time graph back to a position vs. time graph by taking the area under the curve of the velocity graph, which is equal to the distance travelled up to that point.



3. Acceleration (m/s^2)
   a. Acceleration refers to how quickly an object changes velocity, and is modeled by the equation:

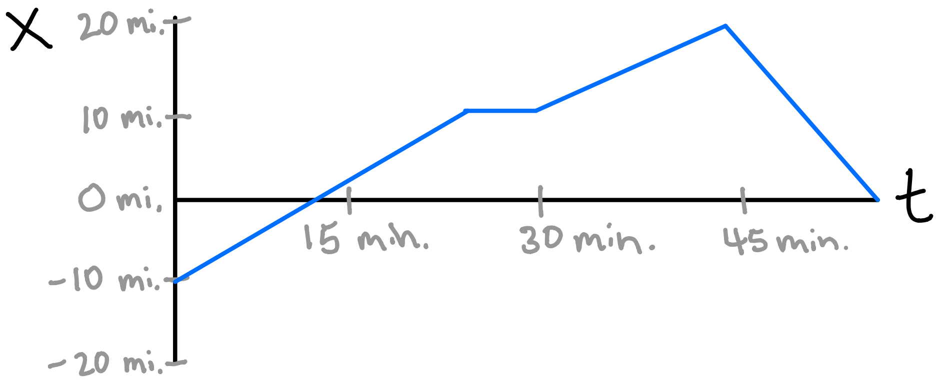
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

   a. Just as the slope of the position vs. time curve equals the velocity, the slope of the velocity vs. time curve equals the acceleration.
   b. Using the velocity vs. time graph to determine the velocity of the object at any given point in time, we can now create an acceleration vs. time graph.
   c. We can also go from an acceleration vs. time graph back to a velocity vs. time graph by taking the area under the curve of the acceleration graph, which is equal to the velocity at that point.

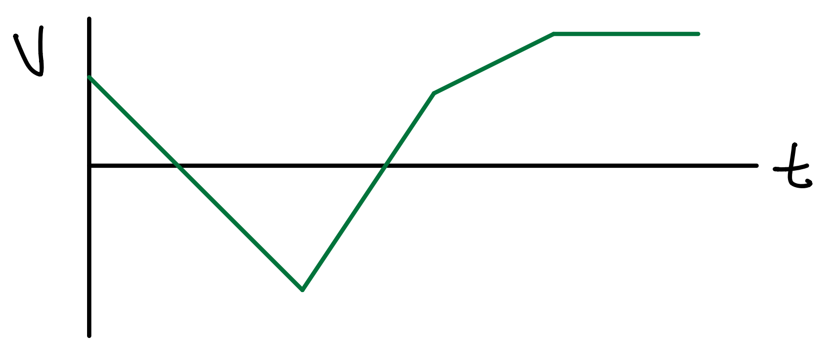


Position, Velocity, and Acceleration Problems

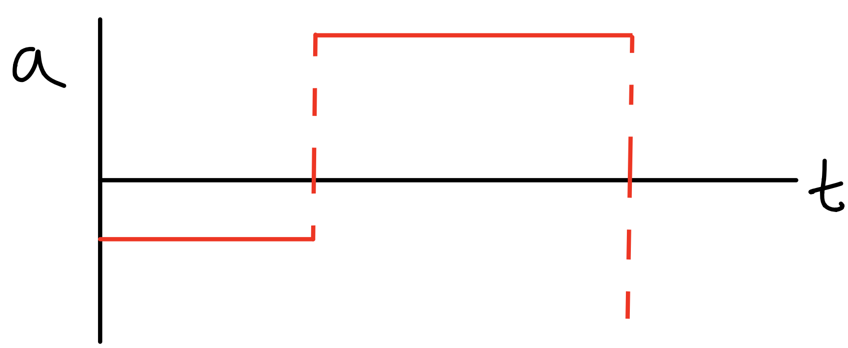
- 1) SEE IT: Below is the position vs. time graph of a car.
    a. What is the total displacement of the car?
    b. What is the total distanced travelled by the car?
    c. What is the average speed of the car over the trip?
    d. What is the instantaneous velocity of the car for the first 25 minutes of the drive?
    e. What is the acceleration of the car 20 minutes into the drive?



- 2) DO IT: Below is the velocity vs. time graph of a figure skater's motion. Draw the position vs. time graph.



- 3) TEACH IT: Below is an acceleration vs. time graph. Draw the position vs. time graph.



1-Demensional Motion

1. The following kinematics equations are derived from $a = \frac{\Delta v}{\Delta t}$
    a. Velocity for an object with constant-acceleration motion:
    $$v_f = v_i + a\Delta t$$
    b. Position for an object with constant-acceleration motion:
    $$x_f = \frac{1}{2}a\Delta t^2 + v_i\Delta t + x_i$$
    c. Relating velocity and displacement for constant-acceleration motion:
    $$v_f^2 = v_i^2 + 2a\Delta x$$
2. In free-fall acceleration, a = g = 9.8 m/s^2

<u>1-Demensional Motion Problems</u>

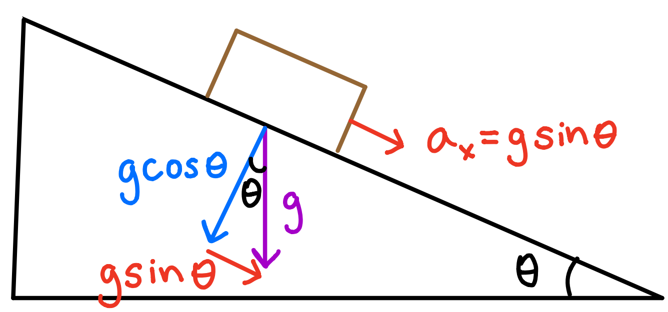
- 4) SEE IT: When the light turns green, a driver stomps on the gas. After accelerating for 94 m, the car is travelling at 32 m/s. Assuming that the car's acceleration is constant over this distance, what is its acceleration?

- 5) DO IT: A dumbbell is dropped from rest off of a 100 m tall skyscraper. Ignoring drag, what is its velocity when it hits the ground?

- 6) TEACH IT: A tennis ball is launched completely vertical with an initial velocity of 12.3 m/s. What is the maximum height the ball can reach?

Ramps

1. Using the kinematics equations, but with one added step
    a. Acceleration due to gravity, g, points straight down and must first be split into its component vectors (vertical and horizontal).
    b. The vertical component, a_y, is perpendicular to the surface of the ramp.
    c. The horizontal component, a_x, is parallel to the surface of the ramp.
    d. Because we are interested in the motion of an object as it moves up or down a ramp (parallel to the ramp's surface), we will use the horizontal component, a_x, in our calculations.
    e. As demonstrated below, $a_x = g\ sin\theta$.



<u>Ramp Problems</u>

- 7) SEE IT: A block starts sliding from rest down a 52° frictionless ramp. How much time passes before the block is travelling at 4 m/s?

- 8) DO IT: On another planet, a block already moving at 3 m/s begins sliding down a 64° frictionless ramp. If the block is travelling 5 m/s after sliding for 12 seconds, what is the ratio of this planet's gravitational constant compared to earth's?

- 9) TEACH IT: A block slides down a 39° frictionless ramp from rest in 6.4 seconds. If the block started sliding from the very top of the ramp, how much horizontal distance would it travel during the course of its motion?

Relative Velocity

1. These questions allow us to find the velocity of $v_{A/C}$ relative to both $v_{A/B}$ and $v_{B/C}$
2. Know your trig and vector rules!
3. To understand this conceptually, it is best to do problems.

<u>Relative Velocity Problems</u>

- 10) SEE IT: A flock of ducks is attempting to fly south for the winter, but a 15 m/s, 20^{o} south of east wind keeps pushing it off course. The flock’s direction of motion relative to earth is 45^{o} south of west, while the direction relative to the air is 5^{o} south of west. What is the flock’s relative speed to an observer standing on the ground?

- 11) DO IT: Cleveland is 300 miles east of Chicago. A plane leaves Chicago flying due east at 500 mph. The pilot is a beginner who does not account for the wind, which is blowing at 100 mph to the south. Draw a diagram of this situation and assign variables to the relative velocities at play.

- 12) TEACH IT: Cleveland is 300 miles east of Chicago. A plane leaves Chicago flying due east at 500 mph. The pilot is a beginner who does not account for the wind, which is blowing at 100 mph to the south. Using your diagram, find the velocity of plane relative to the ground.

Projectiles

1. A projectile is an object in multidimensional motion that is subjected to gravity – a paper ball being thrown into a recycling bin or a basketball travelling towards the rim.
2. Projectile motion is made up of two independent motions – vertical and horizontal motions – so we must first break up the total v_i into its vertical and horizontal components like we did with ramp accelerations.
3. The vertical component of the initial launch velocity, $(v_y)_i$, is affected by gravity and decreases by 9.8 m/s each second. It is mathematically modeled as:
$$(v_y)_i = v_i sin\theta$$
4. The horizontal component of the initial launch velocity, $(v_x)_i$, is unaffected by gravity and remains constant throughout the object's motion. It is mathematically modeled as:
$$(v_x)_i = v_i cos\theta$$
5. In projectile problems, θ is the launch angle.
6. At the motion's peak, $v_y = 0$ m/s.
7. On a flat surface, a projectile will hit the ground with the same speed that it was launched at (note: the sign will be different!).
8. Once again, we will be using the kinematics equations to solve these problems. The equations for horizontal movement are:
$$x_f = x_i + (v_x)_i \Delta t = x_i + v_i cos\theta \Delta t$$
$$(v_x)_i = (v_x)_f = \text{constant}$$
While the equations for vertical movement are:
$$y_f = -\frac{1}{2} g(\Delta t)^2 + \left(v_y\right)_i \Delta t + y_i = -\frac{1}{2} g(\Delta t)^2 + v_i sin\theta \Delta t + y_i$$
$$\left(v_y\right)_f = \left(v_y\right)_i - g\Delta t = v_i sin\theta - g\Delta t$$
9. Most questions will involve using both the horizontal and vertical equations. Start with either, solve for time, and plug into the other to solve the problem.

Projectile Problems

- 13) SEE IT: A golfer hits a drive with a launch angle of 12° and an initial velocity of 47 m/s. What is the maximum height that the ball reaches during its motion?

- 14) DO IT: In 2017, the Florida Gators beat the Tennessee Vols in football because of a last minute "Hail Mary" pass. Gators quarterback Feleipe Franks threw the ball to Tyrie Cleveland, a wide receiver, who caught the ball 66 yards down the field and 5 yards to the left of where Felipe released the ball from. If the ball was in the air for 3.6 seconds and was thrown at an angle of 50° above the horizontal, then what velocity was the football thrown at?

  60.4m

  4.6m

- 15) TEACH IT: A car, starting from rest at the bottom of a 35° ramp, accelerates to 35 m/s in 5.5 s, at which point it launches from the ramp as a projectile. What is the total horizontal distance that the car travels from the time it begins accelerating up the ramp until it hits the ground?

Solutions

1) a. 10 mi.

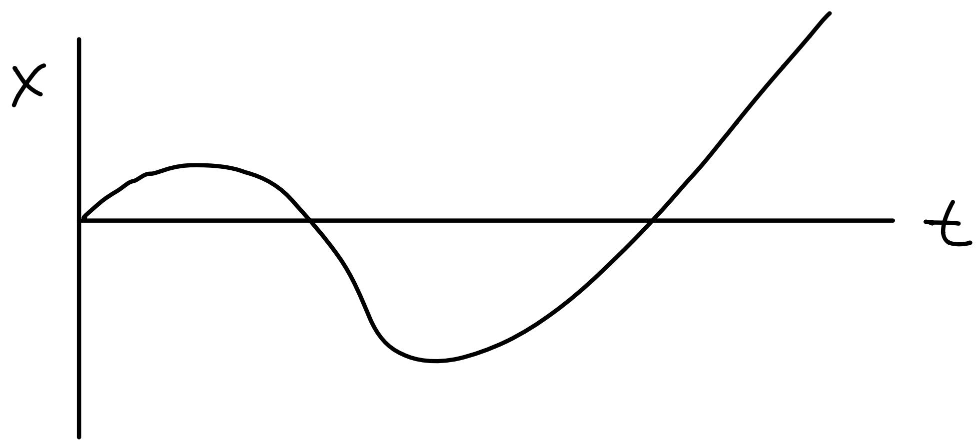
b. 50 mi.

c. $\text{speed}_{\text{ave}} = \dfrac{\text{distance}_{\text{total}}}{\text{time}_{\text{total}}} = 50 \text{ mph}$

d. $v_{\text{inst.}} = m_{x(t)} = \dfrac{20 \text{ mi}}{25 \text{ min}} = 48 \text{ mph}$

e. 0 m/s^2

2)



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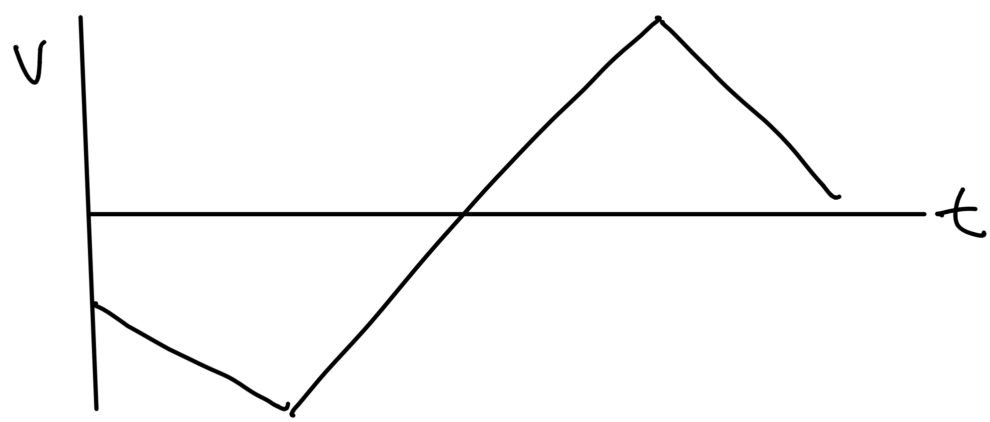
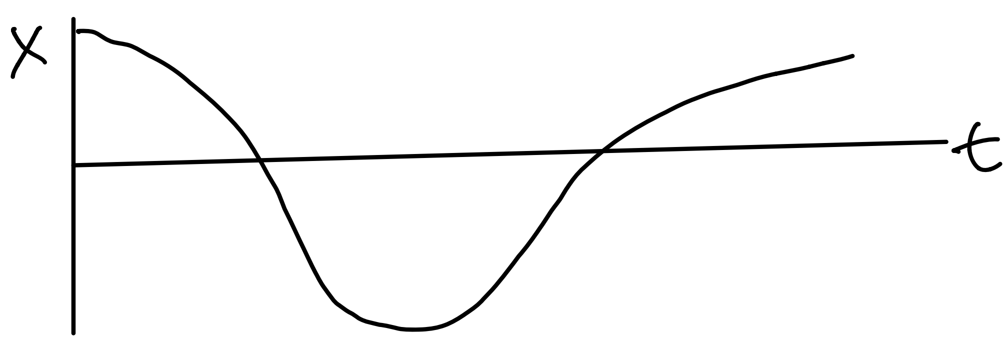




4) $v_f^2 = v_i^2 + 2a\Delta x$

$$a = \frac{v_f^2 - v_i^2}{2\Delta x}$$

$$a = \frac{32^2}{2(94)} = 5.45 \text{ m/s}^2$$

5) $y_F = -\frac{1}{2}gt^2 + \cancel{v_0}^{\,0} t + \cancel{y_i}^{\,0}$

$$-100 = -\frac{1}{2}gt^2$$

$$\sqrt{\frac{-100(-2)}{9.8}} = t = 4.5 \text{ s}$$

$$v_F = -gt + \cancel{v_i}^{\,0}$$

$$v_F = -9.8(4.5) = -44.1 \text{ m/s}$$

6) $\cancel{v_F}^{\,0} = -gt + v_i$

$$t = \frac{v_i}{g} = \frac{12.3}{9.8} = 1.26 \text{ s}$$

$$y_F = -\frac{1}{2}gt^2 + v_i t + \cancel{y_0}^{\,0}$$

$$y_F = -\frac{1}{2}(9.8)(1.26)^2 + 12.3(1.26) = 7.72 \text{ m}$$

7) $V_F = at + \cancel{V_i}^{\,0}$

$t = \frac{V_F}{a} = \frac{4}{9.8\sin(52)} = 0.52\ s$

8) $V_F = at + V_i$

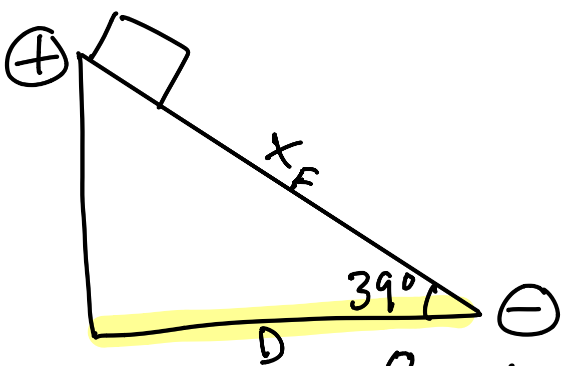
$a = \frac{V_F - V_i}{t}$

$g_p \sin\theta = \frac{V_F - V_i}{t}$

$g_p = \frac{V_F - V_i}{t\sin\theta} = \frac{5-3}{12\sin 64} = 0.19\ m/s^2$

$\frac{g_p}{g_e} = \frac{0.19}{9.81} = 0.02 : 1$

9)



$x_F = -\frac{1}{2}at^2 + \cancel{V_i t}^{\,0} + \cancel{x_i}^{\,0}$

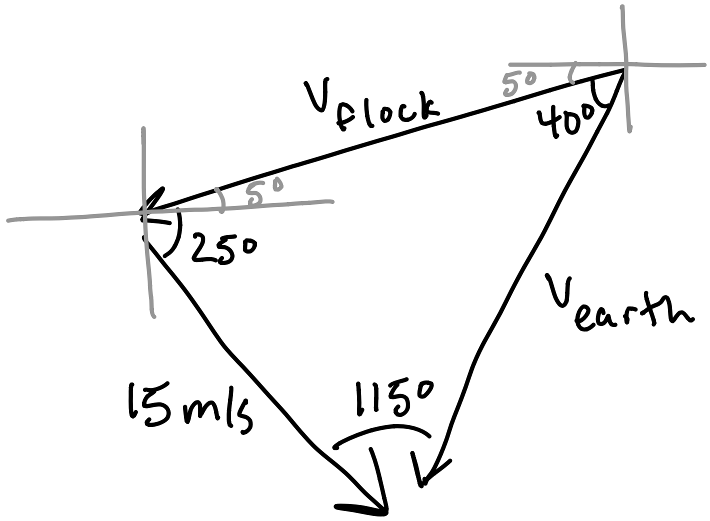
$x_F = -\frac{1}{2}g\sin\theta t^2 = -\frac{1}{2}(9.8)(\sin 39°)(6.4)^2$

$x_F = -126.3\ m$

$\cos\theta = \frac{D}{x_F} \Rightarrow D = x_F\cos\theta = 126.3(\cos 39°)$

$D = 98.15\ m$

10)

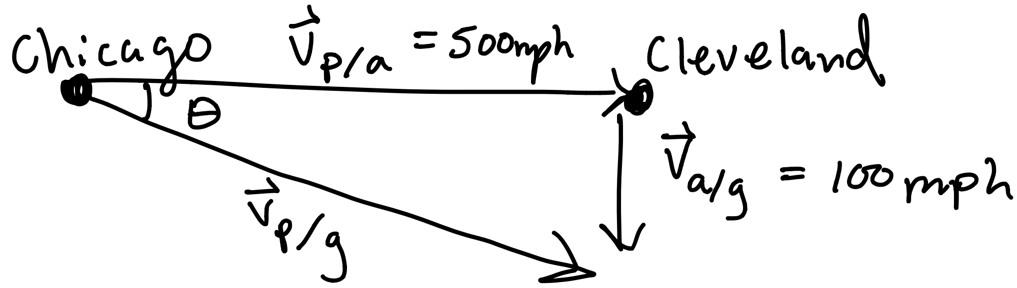


$$\frac{\sin 25°}{V_{earth}} = \frac{\sin 40°}{15}$$

$$V_{earth} = \frac{15 \sin 25°}{\sin 40°}$$

$$V_{earth} = 9.86 \text{ m/s}$$

11)



12) $$\vec{V}_{p/g} = \sqrt{V_{p/a}^2 + V_{a/g}^2} = \sqrt{500^2 + 100^2}$$

$$\vec{V}_{p/g} = 510 \text{ mph}$$

13) $$\cancel{V_F}^{\,0} = -gt + (V_y)_i$$

$$gt = V_i \sin\theta$$

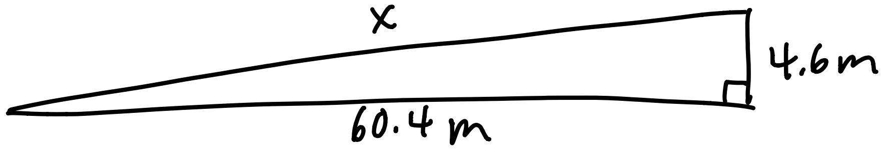
$$t = \frac{V_i \sin\theta}{g} = \frac{47(\sin 12°)}{9.8} = 1.0 s$$

$$y_F = -\frac{1}{2} g \cancel{t^2}^{\,1} + V_i \sin\theta \cancel{t}^{\,1} + \cancel{y_i}^{\,0}$$

$$y_F = -\frac{1}{2} g + V_i \sin\theta$$

$$y_F = -\frac{1}{2}(9.8) + 47(\sin 12°) = 4.87 \text{ m}$$

14)

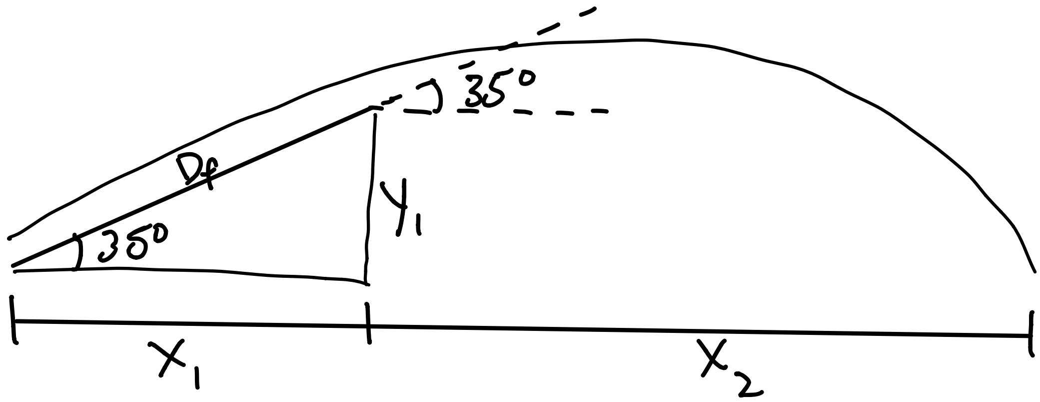


$$x = \sqrt{(60.4)^2 + (4.6)^2} = 60.6\ m$$

$$x = V_i \cos\theta\, t$$

$$V_i = \frac{x}{\cos\theta\, t} = \frac{60.6}{(\cos 50)(3.6)} = 26.18\ m/s$$

15)



$$V_F = at + \cancel{V_i}^{\,0}$$

$$a = \frac{V_F}{t} = \frac{35}{5.5} = 6.36\ m/s^2$$

$$D_f = \frac{1}{2}at^2 + \cancel{V_i t}^{\,0} + \cancel{D_i}^{\,0}$$

$$D_f = 96.195\ m$$

$$\cos\theta = \frac{X_1}{D}$$

$$X_1 = D\cos\theta = 78.8\ m$$

$$\sin\theta = \frac{y_1}{D_F}$$

$$y_1 = D_F \sin\theta$$

$$\cancel{y_F}^{0} = \frac{1}{2}at^2 + v_i t + y_1$$

$$0 = -\frac{1}{2}gt^2 + v_i \sin\theta\, t + D_F \sin\theta$$

$$0 = \left(-\frac{1}{2}\right)(9.8)t^2 + (35\sin 35)t + (96.195\sin 35)$$

$$0 = -4.9t^2 + 20.1t + 55.17$$

Quadratic formula

$$t = 5.98$$

$$x_2 = V_x t = (35\cos 35^\circ)(5.98) = 171.4\text{ m}$$

$$x_1 + x_2 = 78.8\text{ m} + 171.4\text{ m} = 250.2\text{ m}$$