Intermediate Value Theorem

1. Define our function

$$y = f(x)$$

2. Define a y-value

$$m$$

3. Establish that the function is continuous

4. Chose an interval

$$[a, b]$$

5. Establish that m is between $f(a)$ and $f(b)$

6. Conclusion $\longrightarrow$ IVT applies

① Use IVT to prove that the equation $3x^5 - 4x^2 = 3$ is solvable on the interval $[0, 2]$.

1 → $f(x) = 3x^5 - 4x^2$

2 → $m = 3$

3 → ✓

4 → $[0, 2]$

5 → $f(0) = 0$

$f(2) = 3(2)^5 - 4(2)^2 = 80$

$f(0) < m < f(3)$

$0 < 3 < 80$

IVT proves that $f(x) = 3$ on interval

(2) Use the IVT to prove that the equation $x^3 + 2 = \sin x$ is solvable.

$x^3 + 2 - \sin x = 0$

1 → $f(x) = x^3 + 2 - \sin x$

2 → $m = 0$

$[-\pi, 0]$

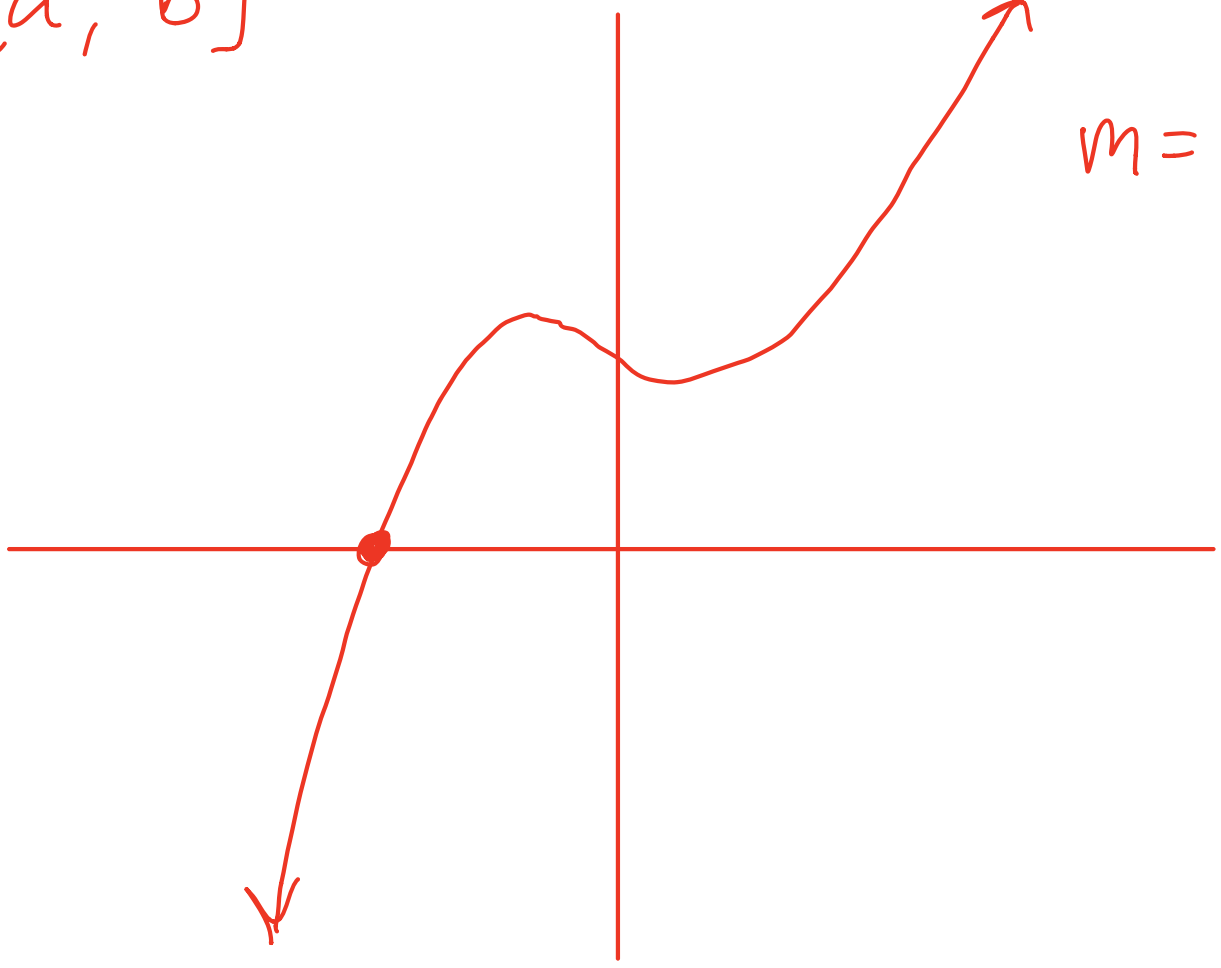
$f(-\pi) < m < f(0)$

$-29 < 0 < 2$

IVT proves that $f(x) = 0$ is on the interval

$[a, b]$

$f(x) = x^3 + 2 - \sin x$

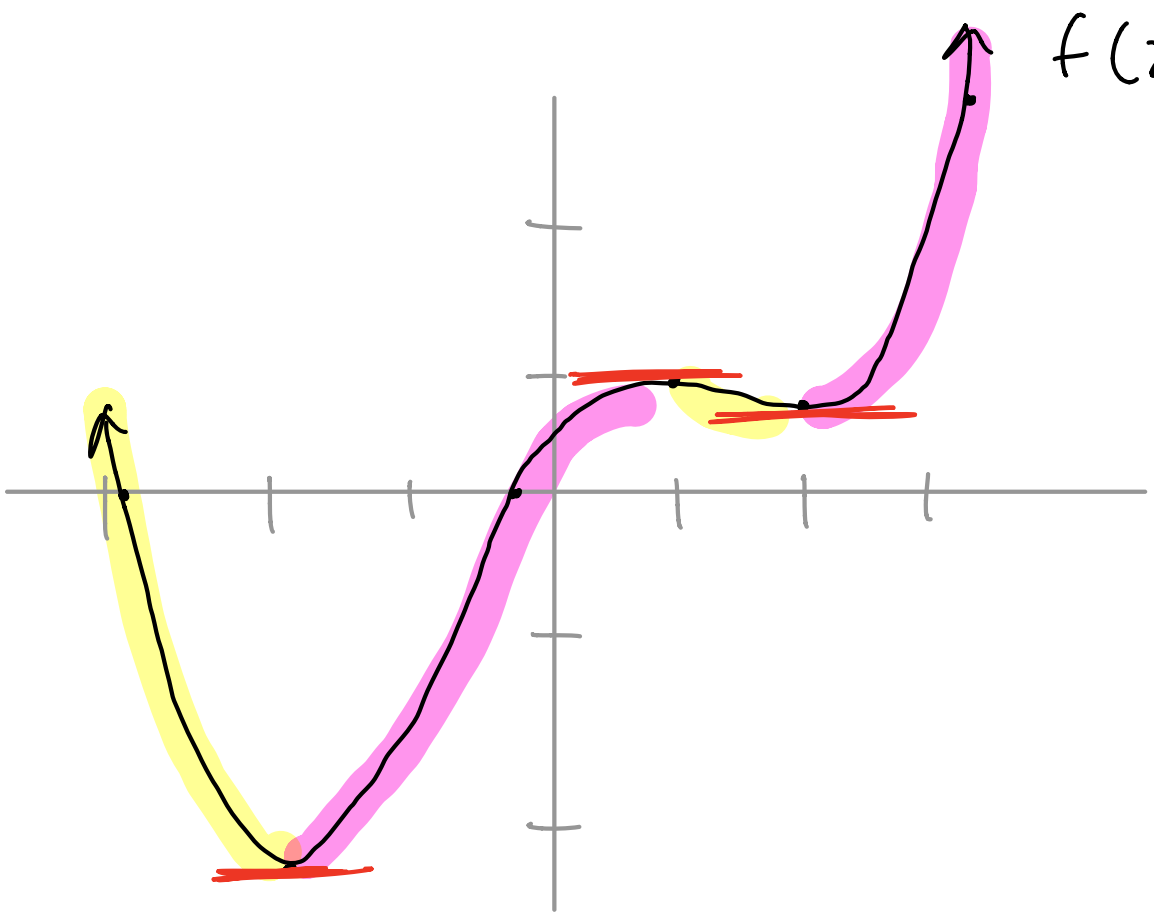


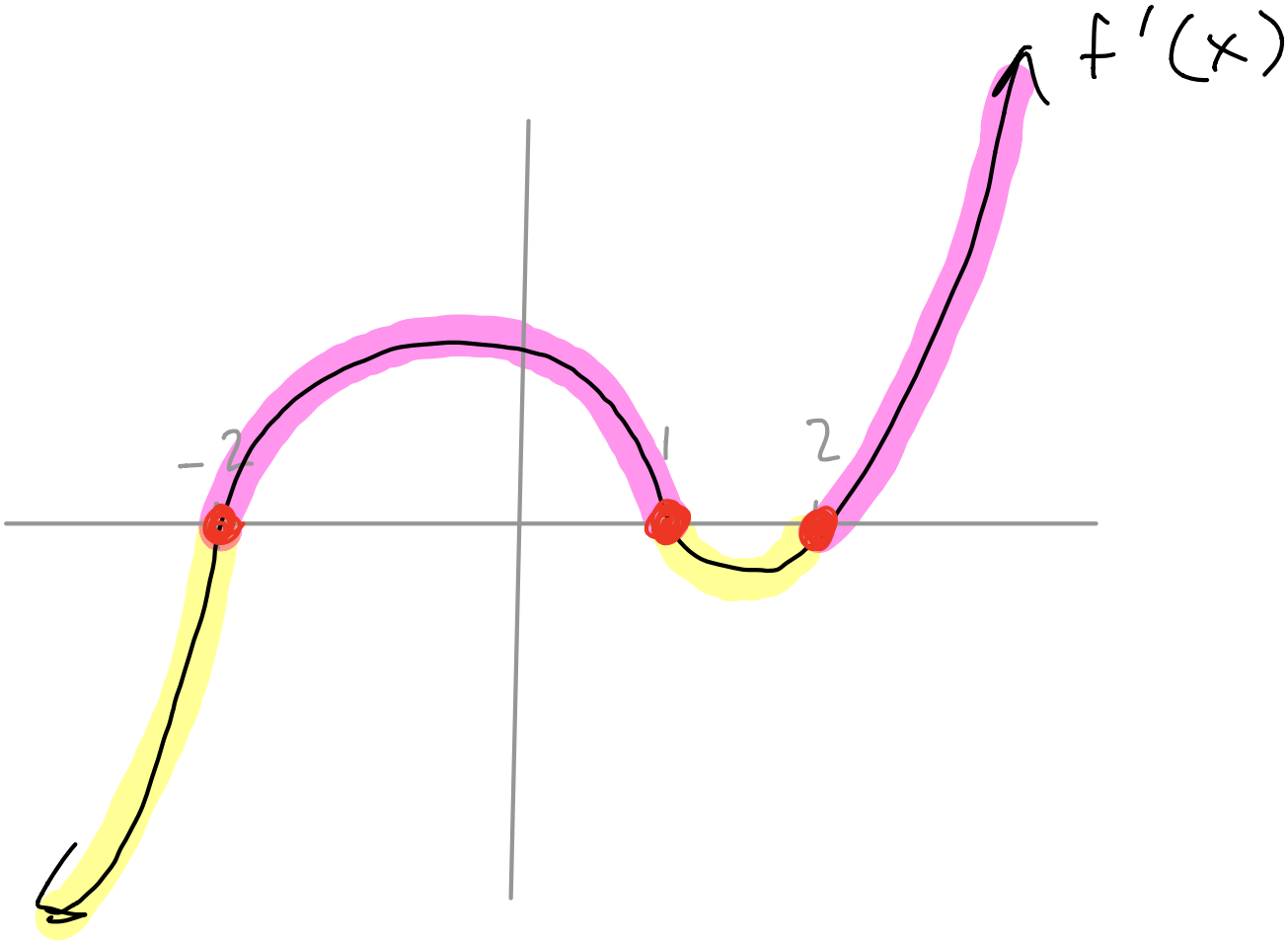
$m = 0$

$[-100, +100]$

$f(-100) < m = 0 < f(100)$

3





(4) $\lim_{t \to -1} \dfrac{t+1}{|t+1|}$

$$|t+1| = \begin{cases} t+1 & t \geq -1 \\ -(t+1) & t < -1 \end{cases}$$

$$\lim_{t \to -1^-} \frac{t+1}{|t+1|} = \lim_{t \to -1^-} \frac{t+1}{-(t+1)} = \lim_{t \to -1^-} -1 = -1$$

$$\lim_{t \to -1^+} \frac{t+1}{|t+1|} = \lim_{t \to -1^+} \frac{t+1}{t+1} = \lim_{t \to -1^+} 1 = 1$$

$$\lim_{t \to -1^-} \neq \lim_{t \to -1^+}$$

Overall limit DNE

Given:

$f'(3) = 2$

$(3, -1)$ is on $f(x)$

Find equation of tan line at $x = 3$

$a = 3$

$y - f(a) = f'(a)(x - a)$

$y - (-1) = 2(x - 3)$

$y + 1 = 2x - 6$

$y = 2x - 7$

$y = mx + b$

c x b

$f(x) = 2x + 1$

↳ $f'(x) = 2 + 0 = 2$

$f(x) = 0x + 1$

↳ $f'(x) = 0 + 0 = 0$