Chain Rule

- Think "layers"
- Work out to in

$$f(x) = (2x^3 + \cos(x))^{50}$$

$$(\qquad\qquad)^{50}$$

$$\times \quad 2x^3 + \cos(x)$$

$$\times \qquad\qquad (x)$$

$$f'(x) = 50(2x^3 + \cos(x))^{49}$$

$$\cdot (6x^2 - \sin x)$$

$$\cdot 1$$

$$f(x) = \cos^4(x) + \cos(x^4)$$

$$= (\cos(x))^4 + \cos(x^4)$$

$$f'(x) = 4(\cos(x))^3 \cdot (-\sin(x)) \cdot 1$$
$$- \sin(x^4) \cdot 4x^3$$

$$f(x) = \sin^5(t^3) + 10$$

$$= (\sin(t^3))^5 + 10$$

$$f'(x) = 5(\sin(t^3))^4 \cdot \cos(t^3) \cdot 3t^2$$

Implicit Diff.

Find y' for $xy=1$.

$$xy = 1$$

$$y = \frac{1}{x} = x^{-1}$$

$$y' = -1x^{-2} = -\frac{1}{x^2}$$

Find the value of the derivative at the point (10, 2) of the function $x^2 + y^2 = 9$

$$x^2 + y^2 = 9$$

$$\frac{d}{dx}\left(x^2 + (y)^2\right) = \frac{d}{dx}(9)$$

$$2x + 2y \cdot y' = 0$$

$$y' = -\frac{x}{y}$$

$$y' = -\frac{10}{2}$$

$$y' = -5$$

Exponential/Logarithmic Diff.

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

$$f(x) = \log_a x$$

$$f'(x) = \frac{1}{x \cdot \ln(a)}$$

$$f(x) = a^x$$

$$f'(x) = a^x \cdot \ln(a)$$

① $R(w) = 4^w - 5\log_9 w$

$$R'(w) = 4^w \cdot \ln(4) - 5 \cdot \frac{1}{w\ln 9}$$

$$= 4^w \ln(4) - \frac{5}{w\ln 9}$$

② $f(x) = 3e^x + \underline{10x^3}\,\underline{\ln x}$

$$f'(x) = 3e^x + 30x^2\ln x + 10x^3\frac{1}{x}$$

$$= 3e^x + 30x\ln x + 10x^2$$

③ $y = \dfrac{5e^x}{3e^x + 1}$

$$y' = \frac{(3e^x+1)(5e^x) - (5e^x)(3e^x)}{(3e^x+1)^2}$$

$$= \frac{\cancel{15e^{2x}} + 5e^x \cancel{- 15e^{2x}}}{(3e^x+1)^2}$$

④ $f(x) = 2e^x - 8^x$

$f'(x) = 2e^x - 8^x \ln(8)$

⑤ $g(t) = 4\log_3(t) - \ln(t)$

$g'(t) = \dfrac{4}{t\ln(3)} - \dfrac{1}{t}$

⑥ $g(x) = e^{1-\cos x}$

$g'(x) = e^{1-\cos x} \cdot \sin x$

⑦ $H(z) = 2^{1-6z}$

$H'(z) = 2^{1-6z} \cdot \ln(2) \cdot -6$

$H'(z) = -6\ln(2)(2^{1-6z})$

(8) $F(y) = \ln(1 - 5y^2 + y^3)$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$F'(y) = \frac{1}{1-5y^2+y^3} \cdot -10y + 3y^2$$

$$F'(y) = \frac{-10y + 3y^2}{1-5y^2+y^3}$$