1. Find the derivative of the function:

$$f(x) = (4t^2 - t)(t^3 - 8t^2 + 12)$$

$$f'(x) = (\text{first})\left(\frac{d}{dx}\text{second}\right) + (\text{second})\left(\frac{d}{dx}\text{first}\right)$$

$$f'(x) = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$$

$$f'(x) = 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

② Find the derivative of the function:

$$y = (1 + \sqrt{x^3})(x^{-3} - 2\sqrt[3]{x})$$

$$y = (1 + x^{\frac{3}{2}})(x^{-3} - 2x^{\frac{1}{3}})$$

$$\frac{dy}{dx} = \left(\frac{3}{2}x^{1/2}\right)\left(x^{-3} - 2x^{1/3}\right) + \left(-3x^{-4} - \frac{2}{3}x^{-2/3}\right)\left(1 + x^{3/2}\right)$$

$$\frac{dy}{dx} = \left(\frac{3}{2}\sqrt{x}\right)\left(\frac{1}{x^3} - 2\sqrt[3]{x}\right) + \left(\frac{-3}{x^4} - \frac{2}{3\sqrt[3]{x^2}}\right)\left(1 + \sqrt{x^3}\right)$$

$$\frac{dy}{dx} = 3x^{-4} - \frac{3}{2}x^{-\frac{5}{2}} - \frac{2}{3}x^{-\frac{2}{3}} - \frac{11}{3}x^{\frac{5}{6}}$$

③ Find the derivative of the function:

$$g(x) = \frac{6x^2}{2-x}$$

$$g'(x) = \frac{(2-x)(12x) - (6x^2)(-1)}{(2-x)^2}$$

$$g'(x) = \frac{24x - 6x^2}{(2-x)^2}$$

④ Find the derivative of the function:

$$R(w) = \frac{3w + w^4}{2w^2 + 1}$$

$l: 2w^2 + 1$

$h': 4w^3 + 3$

$h: w^4 + 3w$

$l': 4w$

$$R'(w) = \frac{(2w^2+1)(4w^3+3) - (w^4+3w)(4w)}{(2w^2+1)^2}$$

$$R'(w) = \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$$

(5) If $f(x) = x^3 g(x)$, $g(-7) = 2$, and $g'(-7) = -9$, determine the value of $f'(-7)$.

$$f'(x) = 3x^2 g(x) + x^3 g'(x)$$

$$f'(-7) = 3(-7)^2(2) + (-7)^3(-9)$$

$$f'(-7) = 3(49)(2) + (-343)(-9)$$

$$f'(-7) = 3381$$

(6) If $f(2) = -8$, $f'(2) = 3$, $g(2) = 17$, and $g'(2) = -4$, determine the value of $(fg)'(2)$.

$$y = f(x) \cdot g(x)$$

$$y' = (fg')(x)$$

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$(fg)'(2) = (3)(17) + (-8)(-4)$$

$$(fg)'(2) = 83$$

(7) Find the equation of the tangent line to $f(x) = (1 + 12\sqrt{x})(4 - x^2)$ at $x = 9$.

$$y = \underline{f(9)} + \underline{f'(9)}(x - 9) \checkmark$$

1) Evaluate $f'(x)$

2) Plug in $x = 9$ to solve $f'(9)$

3) Plug in $x = 9$ to solve $f(9)$

4) Plug in 2) and 3) into tan. line equation

5) Rearrange into $y = mx + b$

$$f'(x) = (6x^{-1/2})(4-x^2) + (1+12\sqrt{x})(-2x)$$

$$f'(x) = \left(\frac{6}{\sqrt{x}}\right)(4-x^2) + (1+12\sqrt{x})(-2x)$$

$$f'(9) = \left(\frac{6}{\sqrt{9}}\right)(4-9^2) + (1+12\sqrt{9})(-2(9))$$

$$f'(9) = ((2)(-77)) + ((37)(-18))$$

$$f'(9) = -820$$

$$f(9) = (1+12\sqrt{9})(4-(9)^2)$$

$$f(9) = 37(-77)$$

$$f(9) = -2849$$

$$y = f(9) + f'(9)(x-9)$$

$$y = -2849 - 820(x-9)$$

$$y = -2849 - 820x + 7380$$

$$y = -820x + 4531$$

(8) Determine where

$$f(x) = \frac{x - x^2}{1 + 8x^2}$$ is increasing and decreasing.

$$f'(x) = \frac{(1-2x)(1+8x^2) - (x - x^2)(16x)}{(1+8x^2)^2}$$

$$f'(x) = \frac{1 - 2x - 8x^2}{(1+8x^2)^2}$$

denom. doesn't equal zero for any real value

$$1 - 2x - 8x^2 = 0$$

$$-(4x-1)(2x+1) = 0$$

$x = -\frac{1}{2}$ and $\frac{1}{4}$

$f'(-1) = \ominus$ $\therefore$ dec. $f'(0) = \oplus$ $\therefore$ inc. $f'(1) = \ominus$ $\therefore$ dec.

Increasing: $-\frac{1}{2} < x < \frac{1}{4}$

Decreasing: $-\infty < x < -\frac{1}{2}$,

$\frac{1}{4} < x < \infty$