Assumption: $f(c)$ exists

Critical points on $f(x)$

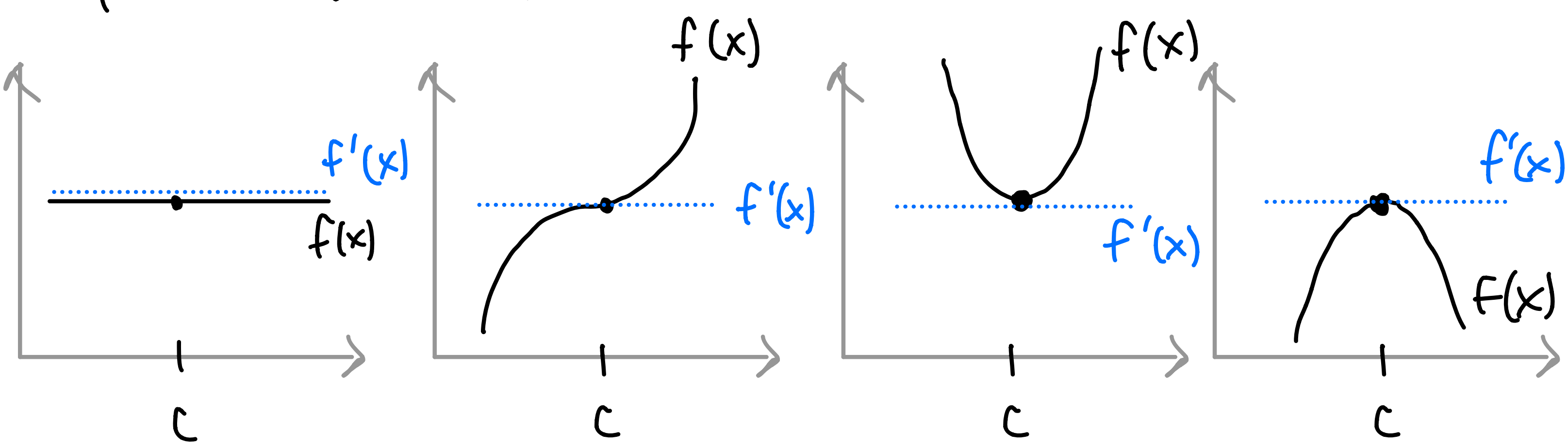
$$f'(c) = 0$$

Constant function

inflection point

local min.

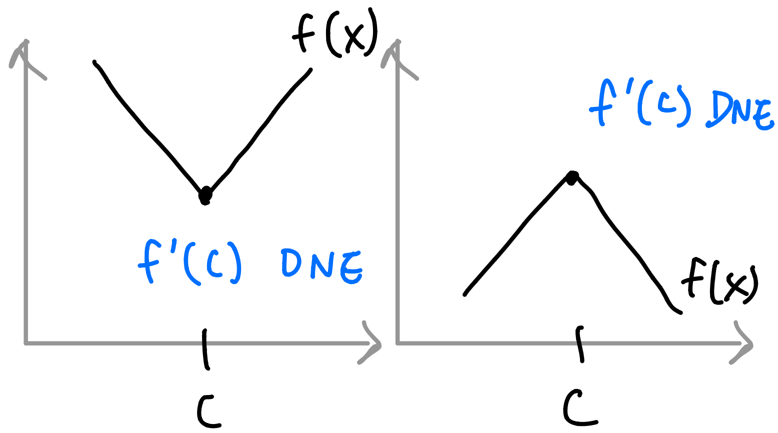
local max.



$f'(c)$ DNE

local min.

local max.



Then we have a critical point at $x=c$

Determine the critical points for the function

$$f(x) = 6x^5 + 33x^4 - 30x^3 + 100$$

$$f'(x) = 30x^4 + 132x^3 - 90x^2 = 0$$

$$f'(x) = 6x^2(5x^2 + 22x - 15) = 0$$

$$f'(x) = 6x^2(5x-3)(x+5) = 0$$

$6x^2 = 0$ $\quad$ $5x - 3 = 0$ $\quad$ $x + 5 = 0$

$x = 0$ $\quad$ $5x = 3$ $\quad$ $x = -5$

$$x = \frac{3}{5}$$

Determine the critical points for the function

$$R(w) = \frac{w^2+1}{w^2-w-6}$$

$$R'(w) = \frac{-w^2-14w+1}{(w^2-w-6)^2}$$

$$R'(w) = \frac{-w^2-14w+1}{((w-3)(w+2))^2} = 0$$

$w-3=0$ $\qquad\qquad$ $w+2=0$

$\cancel{w=3}$ $\qquad\qquad$ $\cancel{w=-2}$

$a=1$

$b=14$

$c=-1$

$$w^2+14w-1=0$$

$$w=\frac{-b\pm\sqrt{b^2-4(a)(c)}}{2a}$$

$$w=\frac{-14\pm\sqrt{14^2-4(1)(-1)}}{2(1)}$$

$$w=\frac{-14\pm\sqrt{200}}{2}$$

$$w=\frac{-14\pm10\sqrt{2}}{2}$$

$$w=-7\pm5\sqrt{2}$$

$w=-7+5\sqrt{2}$ $\qquad$ $w=-7-5\sqrt{2}$

1) Differentiate

2) Determine where $f'(x)$ DNE

3) Determine where $f'(x) = 0$

4) Check to make sure that values from 2) and 3) exist on $f(x)$

Identify the absolute and relative extrema for the following

$$f(x) = x^2 \quad \text{on } [-2, 2]$$

$$f'(x) = 2x = 0$$

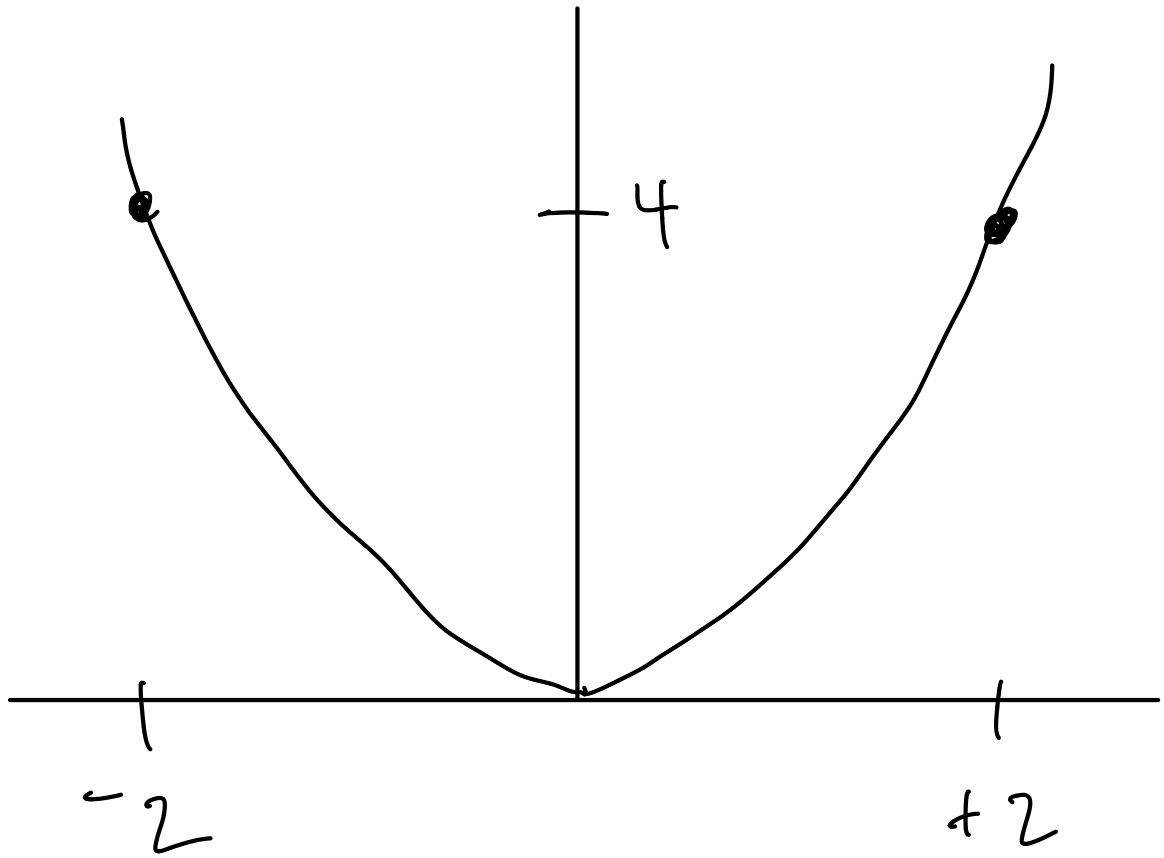
$$2x = 0$$

$$x = 0$$

$$f(0) = 0^2 = 0$$

$$f(-2) = (-2)^2 = 4$$

$$f(2) = (2)^2 = 4$$


4
-2
+2

Identify the inflection points, absolute extrema, and relative extrema for

$$f(x) = x^3$$

$$f'(x) = 3x^2 = 0$$

$$x = 0$$

Determine the absolute extrema for the following:

$$g(t) = 2t^3 + 3t^2 - 12t + 4 \qquad [0, 2]$$

$$g'(t) = 6t^2 + 6t - 12$$

$$g'(t) = 6(t^2 + t - 2)$$

$$6(t+2)(t-1) = 0$$

$t + 2 = 0$ $\quad$ $t - 1 = 0$

$t = -2$ $\quad$ $t = 1$

$g(0) = 4$ $\quad$ $t = 2$ abs. max.

$g(1) = -3$ $\quad$ $t = 1$ abs. min.

$g(2) = 8$

If ...

1) $f(x)$ is continuous on closed intecval $[a, b]$

2) $f(x)$ is differentiable on open intecval (a, b)

Then ...

1) There is a number c such that $a < c < b$

2) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$$f'(x) = m_{tan} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

Determine all the numbers c which satisfy the conclusions of the MVT:

a b

$$f(x) = x^3 + 2x^2 - x \qquad [-1, 2]$$

$$f'(x) = 3x^2 + 4x - 1$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$3c^2 + 4c - 1 = \frac{(2^3 + 4^2 - 2) - (-1^2 - 2^2 + 1)}{2 + 1}$$

$$3c^2 + 4c - 1 = \frac{14 - 2}{3} = \frac{12}{3} = 4$$

$$3c^2 + 4c - 1 = 4$$

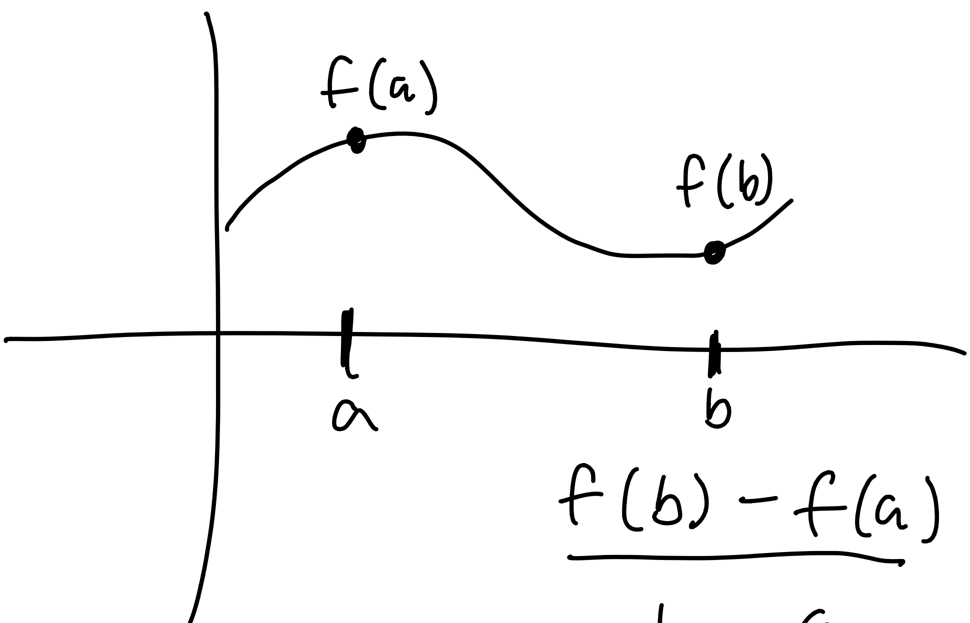
$$3c^2 + 4c - 5 = 0$$

$$C = \frac{-4 \pm \sqrt{16 - 4(3)(-5)}}{6}$$

$$C = \frac{-4 \pm \sqrt{76}}{6}$$

$$C = \frac{-4 + \sqrt{76}}{6} \qquad C = \frac{-4 - \sqrt{76}}{6}$$

$$C = 0.786 \qquad \cancel{C = -2.11}$$



$$\frac{f(b) - f(a)}{b - a}$$

If:

1) $f(x)$ is continuous on closed interval $[a, b]$

2) $f(x)$ is differentiable on open interval (a, b)

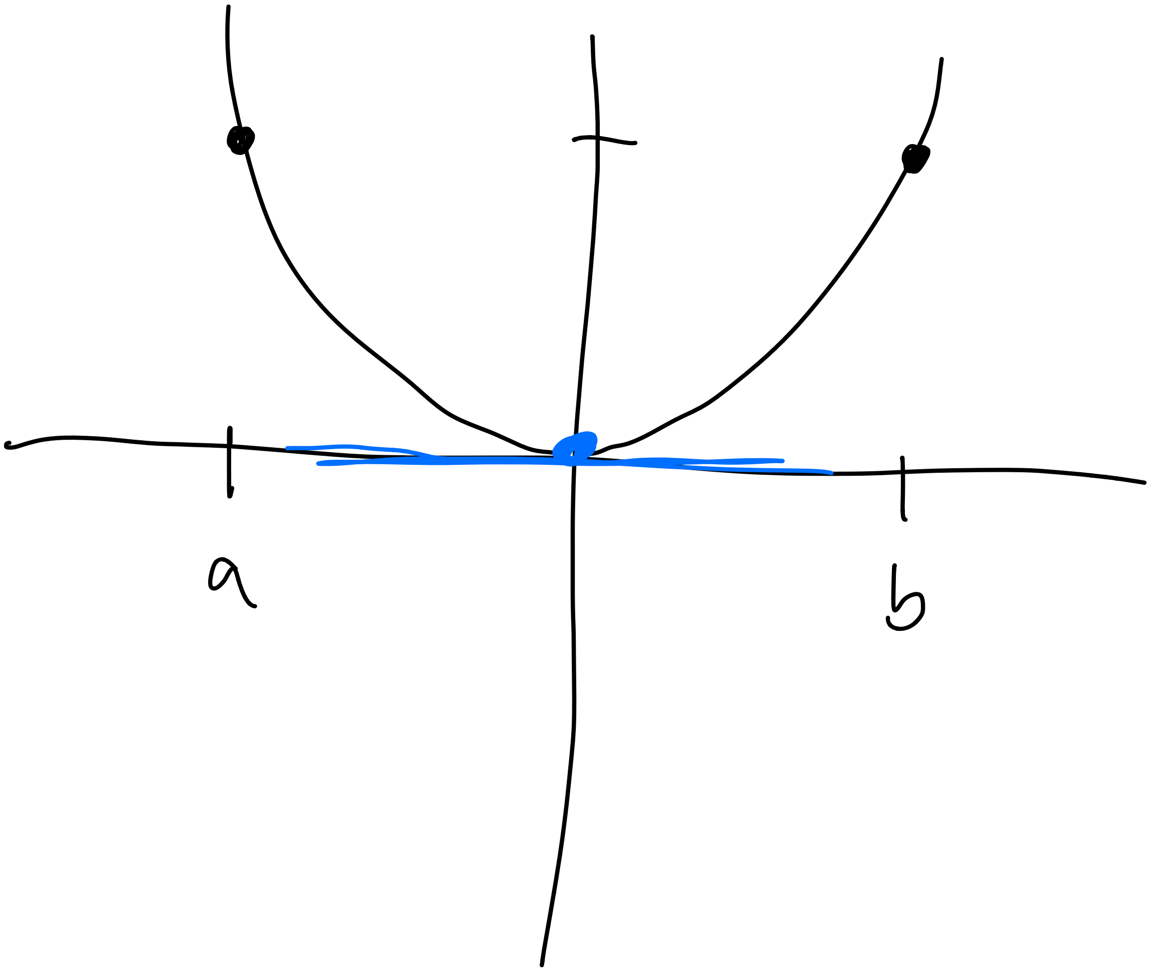
3) $f(a) = f(b)$

Then:

There is a number c such that $a < c < b$ and $f'(c) = 0$

Basically... $f(x)$ has a critical point on the interval

$y = x^2$



Show that the following has exactly one real root:

$$f(x) = 4x^5 + x^3 + 7x - 2$$

$f(0) = -2$

$f(0) < 0 < f(1)$

$f(1) = 10$

$f(a) = f(b)$

$f'(c) = 0$

$$f'(x) = 20x^4 + 3x^2 + 7$$