Pre-Health Post-Baccalaureate Program

PHY2053 Study Guide & Practice Problems

Topics Covered:

Impulse and Momentum
Conservation of Momentum
Angular Momentum

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Impulse and Momentum

Impulse describes a collision, when a force acts upon an object for a short period of time. In a *F-t* graph, impulse is the area under the curve. Impulse (*J*) is given by the following:

$$J = F_{ave}\,\Delta t$$

Momentum is the resulting change in the object's mass x velocity because of the collision. Because it is a vector quantity, momentum must be broken up into its x and y-components. Momentum (*p*) is given by the following:

$$\vec{p} = m\vec{v}$$

The Impulse-Momentum Theorem states that impulse = momentum. Let's take a step back and revisit Newton's Second Law to understand why:

$$\vec{F}_{ave} = m\,\vec{a}_{ave}$$

Rearrange:

$$\vec{a}_{ave} = \frac{\vec{F}_{ave}}{m}$$

Using the definition of acceleration, we also get:

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t}$$

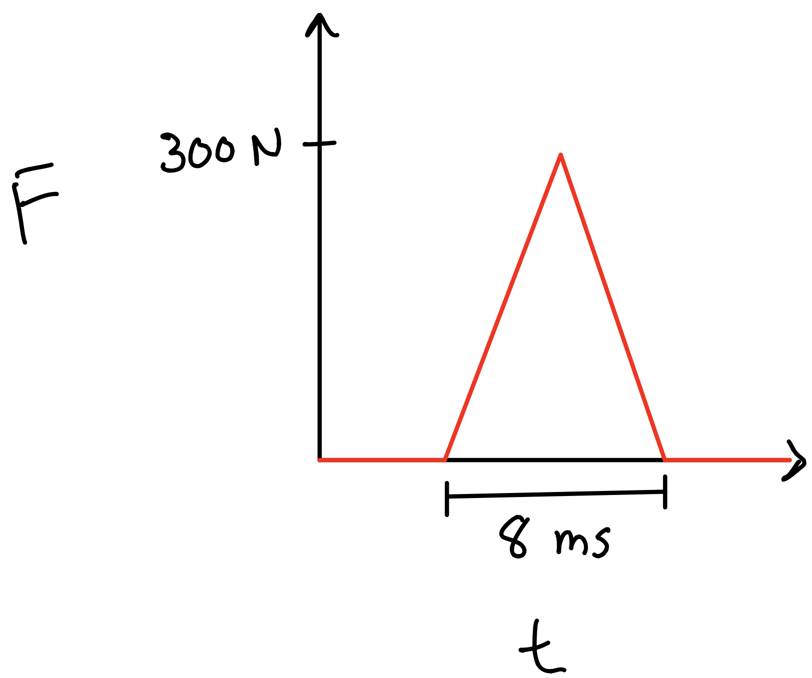
Set the two expressions for acceleration equal to each other:

$$\frac{\vec{F}_{ave}}{m} = \frac{\Delta\vec{v}}{\Delta t}$$

Rearrange:

$$\vec{F}_{ave}\,\Delta t = m\,\Delta\vec{v}$$

1) The following *F-t* graph describes a foot's collision with a 450g soccer ball which is at rest. How fast is the ball traveling after being kicked?



Law of Conservation of Momentum

Newton's Third Law tells us that each force has an equal and opposite force (the two forces form an action/reaction pair). That means, during a collision, the force of object one on object two equals the force of object two on object one. Because they are colliding with each other, the collision time is the same for both objects.

Because the two objects involved in the collision have the same force and time, the impulse is the same, but in opposite directions, for the objects involved in the collision. Since impulse equals momentum, the momentum subject to internal forces is conserved:

$$(P_{1_x})_f + (P_{2_x})_f + (P_{3_x})_f = (P_{1_x})_i + (P_{2_x})_i + (P_{3_x})_i$$

2) Two ice skaters, Sue and Ann, stand facing each other on ice (ignore friction). Sue has a mass of 45kg and Ann has a mass of 80kg. They push off each other, so that they begin to skate backwards and away from each other. Sue has a speed of 2.2 m/s after the push. What is Ann's speed?

Angular Momentum

Just as we previously formulated angular equations similar to already established linear equations, we can similarly develop an angular expression of impulse and momentum:

Linear

$$\vec{F}_{net}\Delta t = m\Delta\vec{v}$$

Angular

$$\tau_{net}\,\Delta t = I\omega$$

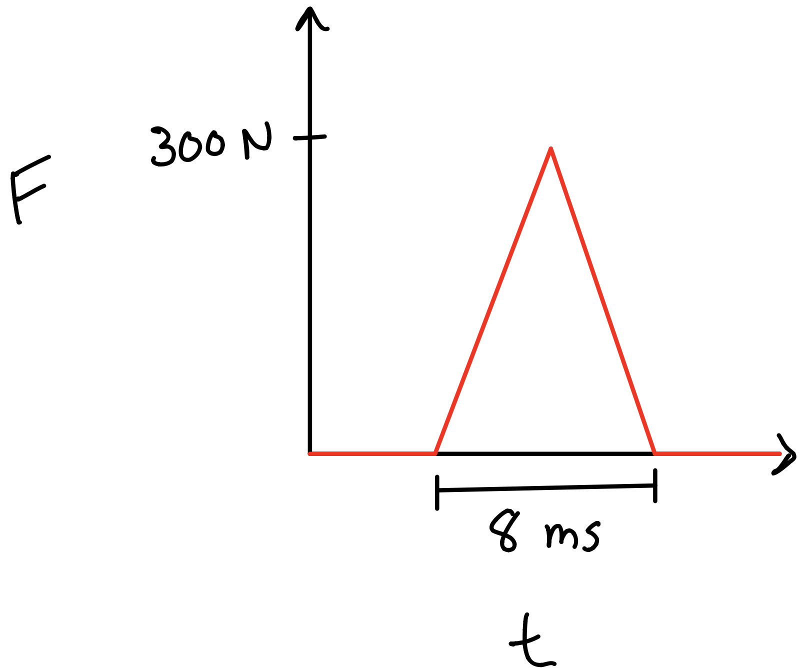
Torque is the angular equivalent of force, moment of inertia is the angular equivalent of mass, and angular velocity is the angular equivalent of velocity.

Angular systems are subject to the Law of Conservation of Angular Momentum.

3) A 36kg man (I = mR^2) stands at the center of a 200kg merry-go-round (I = 1/2 MR^2) which completes a full rotation every 2.5s. While the merry-go-round is rotating, the man walks 2m to the outer edge. Once the man gets to the outer edge, what is the new period of the merry-go-round?

Solutions

1



$F = 300\,N$

$\Delta t = .008\,s$

$m = .450\,kg$

$$J = p$$

$$\text{area} = mv$$

$$v = \frac{\text{area}}{m}$$

$$v = \frac{\frac{1}{2}\Delta t\,\vec{F}_{ave}}{m}$$

$$v = \frac{(\frac{1}{2})(.008)(300)}{.450\,kg}$$

$$\boxed{v = 2.67 \quad m/s}$$

2

$$(p_S)_f + (p_A)_f = \cancelto{0}{(p_S)_i} + \cancelto{0}{(p_A)_i} = 0$$

$$(p_S)_f = -(p_A)_f$$

$$(45)(2.2) = -80v$$

$$v = \frac{45(2.2)}{-80}$$

$$v = -1.24 \text{ m/s}$$

$$\boxed{\text{speed} = 1.24 \text{ m/s}}$$

③ $(L_{man})_i + (L_{mer})_i = (L_{man})_f + (L_{mer})_f$

$$\cancelto{0}{(I_m \omega_m)_i} + (I_m \omega_m)_i = (I_m \omega_m)_f + (I_m \omega_m)_f$$

$$0 + \left(\tfrac{1}{2}MR^2\omega\right)_i = \left(mR^2\omega\right)_f + \left(\tfrac{1}{2}MR^2\omega\right)_f$$

$$0 + \left(\tfrac{1}{2}MR^2\omega\right)_i = \omega_f\left[(mR^2)_f + \left(\tfrac{1}{2}MR^2\right)_f\right]$$

$$\omega_f = \frac{\left(\cancel{\tfrac{1}{2}}M\cancel{R^2}\omega\right)_i}{(2m\cancel{R^2})_f + \left(\cancel{\tfrac{1}{2}}M\cancel{R^2}\right)_f} \cdot \frac{\cancel{2}}{\cancel{2}}$$

$$\omega_f = \frac{M\omega_i}{2m + M}$$

$$\omega_f = \frac{200\left(\frac{2\pi}{2.5}\right)}{2(36) + 200} = 1.85 \text{ rad/s}$$

$$D = rt$$

$$2\pi = \omega t$$

$$t = \frac{2\pi}{\omega} = \frac{2\pi}{1.85} = \boxed{3.4 \text{ s}}$$